[This question paper contains 4 printed pages.]

**1462-A** *Your Roll No.* ............

**B.A./B.Sc. (Hons.)/I** **A**

MATHEMATICS – Unit III

(Analysis – I)

(Admissions of 2008 and before)

*Time : 2 Hours* *Maximum Marks : 38*

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

*Attempt any* **two** *parts from each question.*

## SECTION I

1. (a) Prove that for all real numbers x and y and $\in > 0$

   (i) $|x-y| < \in \Leftrightarrow y - \in < x < y + \in$

   (ii) $|x+y|^2 + |x-y|^2 = 2(|x|^2 + |y|^2)$ (5)

   (b) Define neighbourhood of a point. Show that a set N is a neighbourhood of a point p if and only if there exists a positive integer n such that

$$]p - \frac{1}{n}, p + \frac{1}{n}[ \subset N$$ (5)

P.T.O.

(c) (i) Prove that set of integers has no limit point.

(ii) Prove that the derived set of every set is a closed set. (5)

## SECTION II

2. (a) Define a Cauchy sequence. Prove that every Cauchy sequence is Convergent. Is the Converse true ? (5)

(b) Let <$a_n$> be a sequence defined by

$$a_{n+1} = \frac{1}{2}\left(a_n + \frac{2}{a_n}\right), \; a_1 > 0$$

Show that <$a_n$> is a bounded monotonic sequence. Find the limit of this sequence. (5)

(c) (i) Define limit superior and limit inferior of a bounded sequence. Find these for the sequence <$a_n$>, where

$$a_n = (-1)^n\left(1 + \frac{1}{n}\right), \; n \in \mathbb{N}$$

(ii) If the limit inferior of a sequence <$a_n$> is m, then prove that no subsequence of <$a_n$> can converge to a limit less than m. (5)

## SECTION III

3. (a) Prove that a positive term series converges if and only if the sequence of its partial sums is bounded above. Use this result to show that the positive term geometric series $\sum_{n=0}^{\infty} r^n$ converges if $0 < r < 1$ and diverges if $r \geq 1$. (5)

(b) State Cauchy's integral test for the convergence of an infinite series of positive terms. Use it to prove that $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$. (5)

(c) Test the convergence of any **two** of the following :

(i) $\sum_{n=1}^{\infty} \frac{1}{2^n + x}$, $x \geq 0$

(ii) $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} \cdot \frac{1}{2n+1}$

(iii) $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n\alpha}{n^3}$, $\alpha$ being real. (5)

## SECTION IV

4. (a) If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0. \quad (4)$$

P.T.O.

(b) If $z = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, then show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \sin 2z \qquad (4)$$

(c) Sketch the graph of the curve

$$y(1 + x^2) = x \qquad (4)$$